Real-Time Scheduling for Control Systems

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RTNS09, Paris, France, 26-27 October 2009

Aim of this talk

- to provide on overview of real-time and control
- to stimulate new ideas in the audience
- to have at least one who does not fall alseep

Overview

Part I

Analysis and Design of Real-Time Systems

Part II

Issues in Control Systems

Part III

Ideas for the Future

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Ideas for the Future

The job of the real-time researcher

To play with deadlines, priorities, computation times, periods,...

until the application performs well

Real-Time parameters

Real-Time parameters are cathegorized in

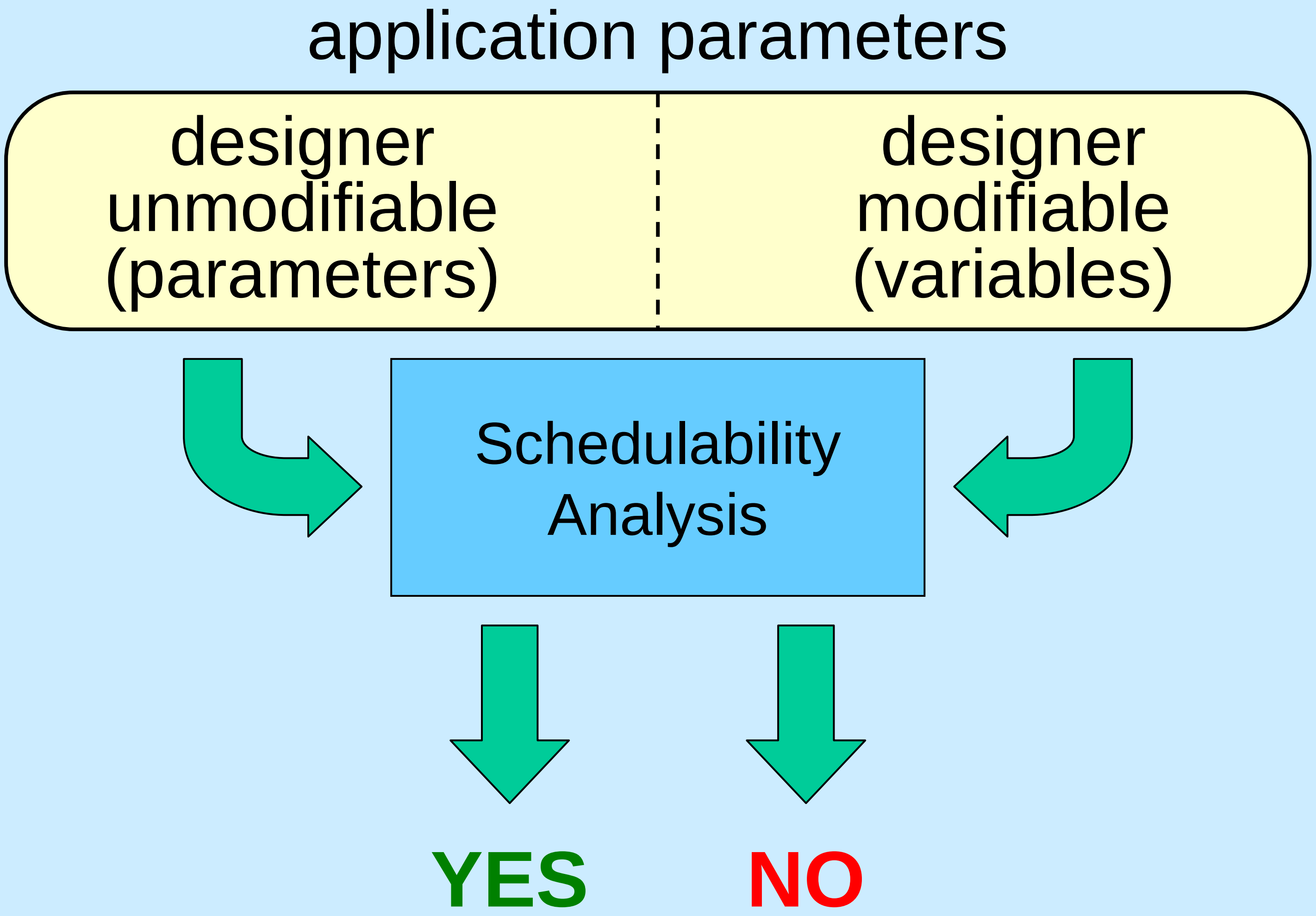
designer unmodifiable (**parameters**)

- activation from an external interrupt
- computation times (C_i) of stand alone code

designer modifiable (**variables**)

- priority of tasks
- deadlines (D_i)
- periods (T_i) in timer driven tasks

Schedulability Analysis

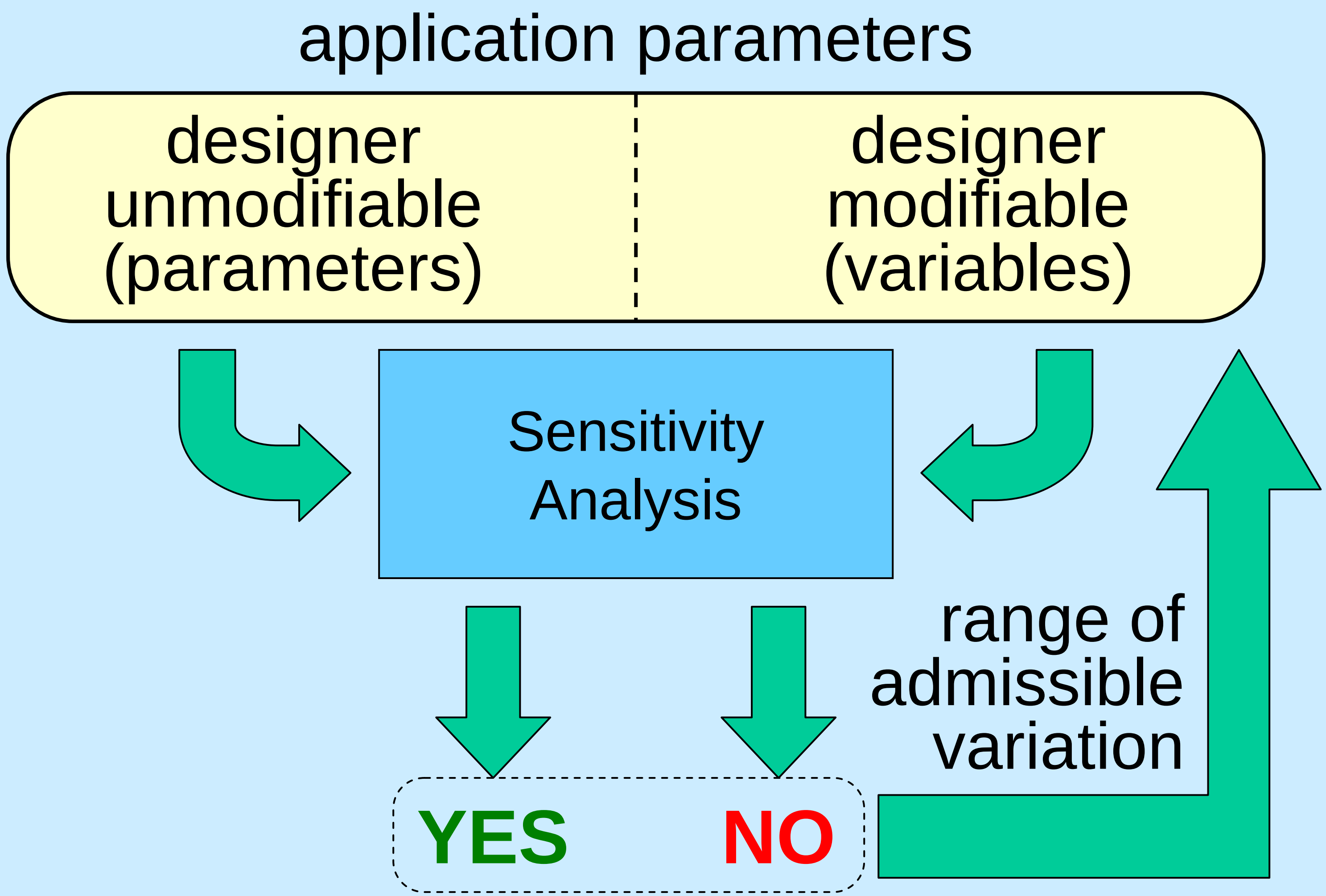


Schedulability Analysis

Requires all variables to be set in advance

- application developer must set all variables (priorities, deadlines,...) in advance
- real-time analyst can apply the preferred schedulability analysis

Sensitivity Analysis

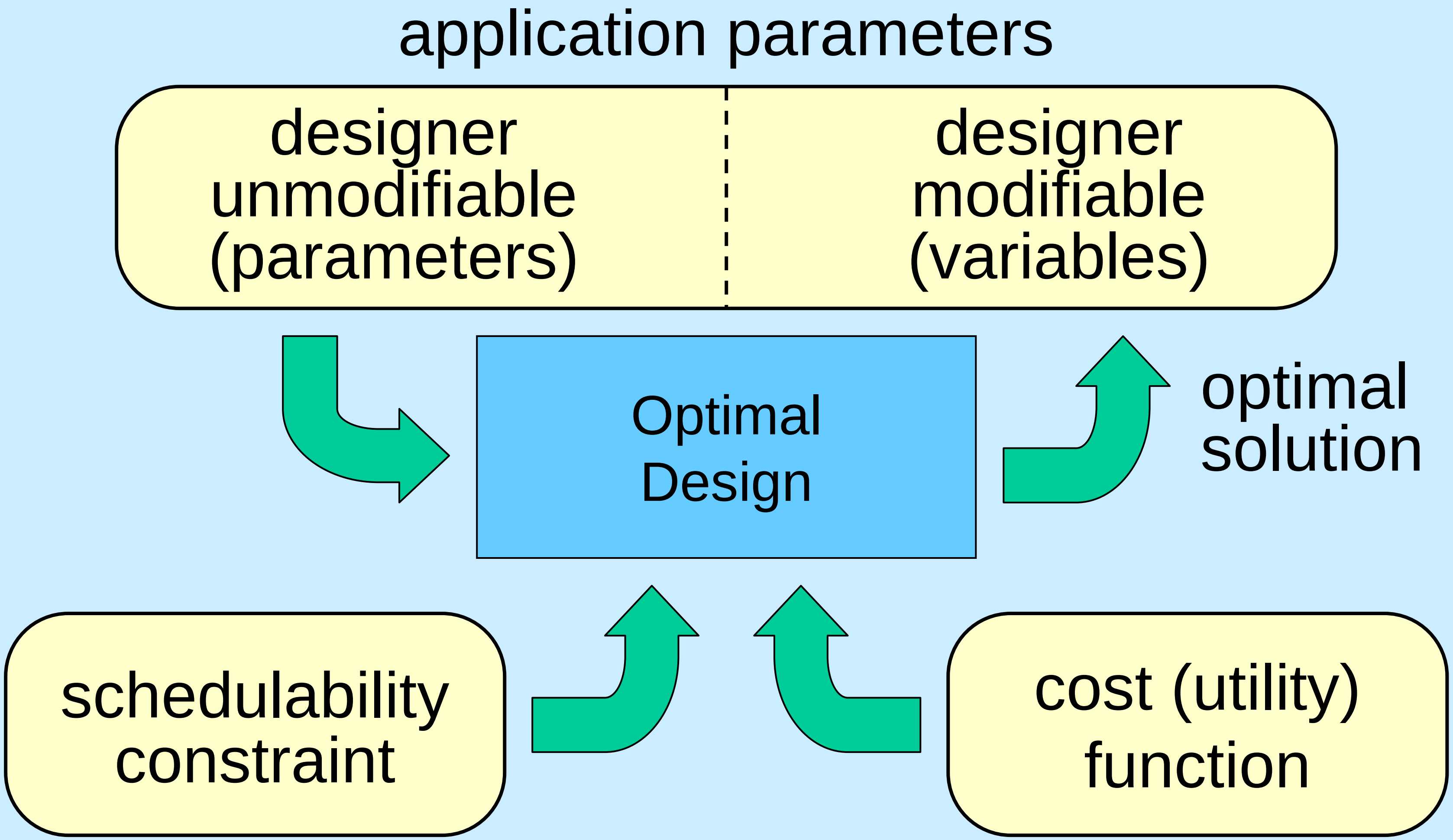


Sensitivity Analysis

Requires only parameters to be set in advance

- application developer must set an initial guess of all variables
- real-time analyst responds with the range of admissible variation of the variables

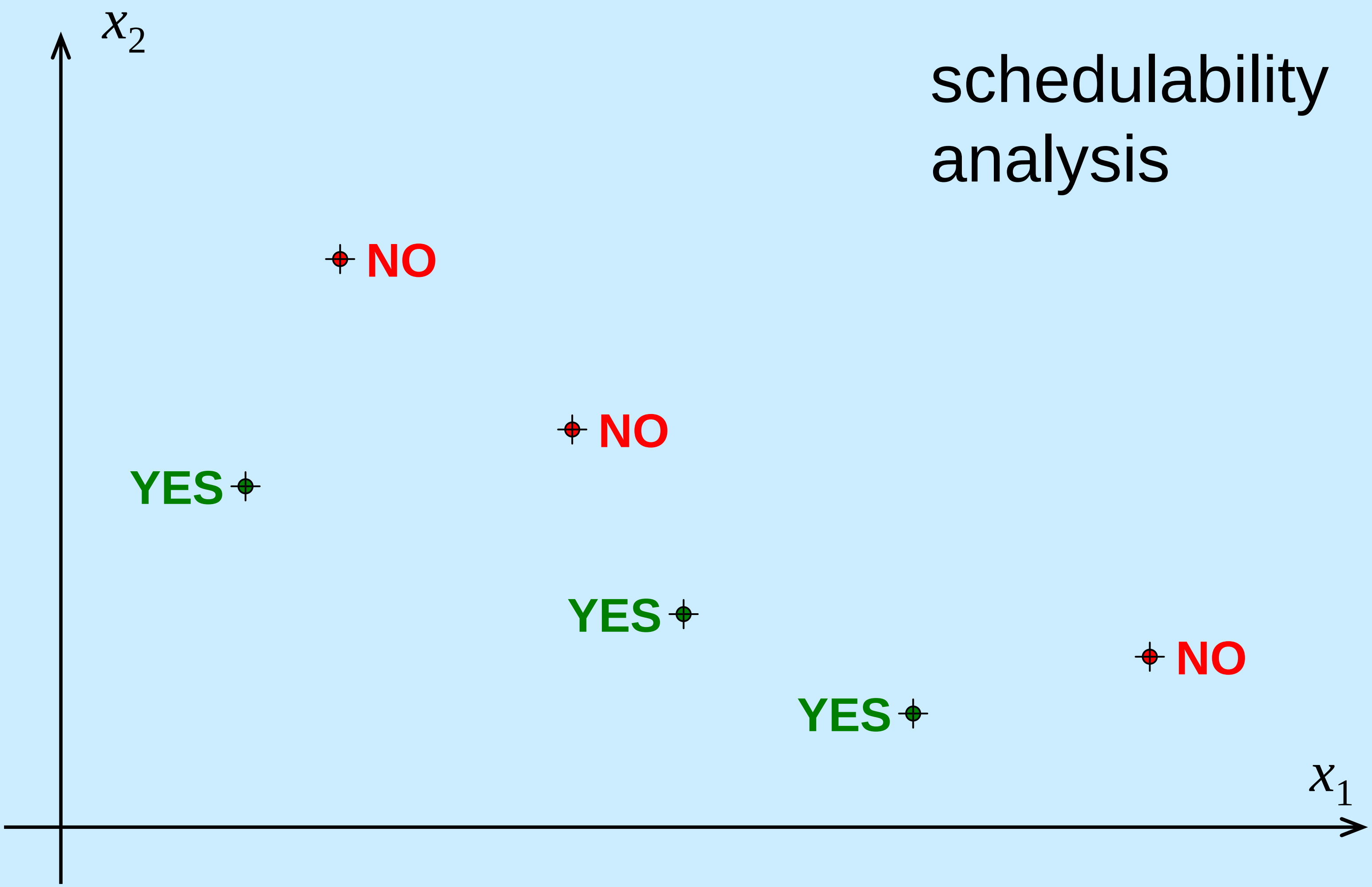
Optimal Design



Optimal Design

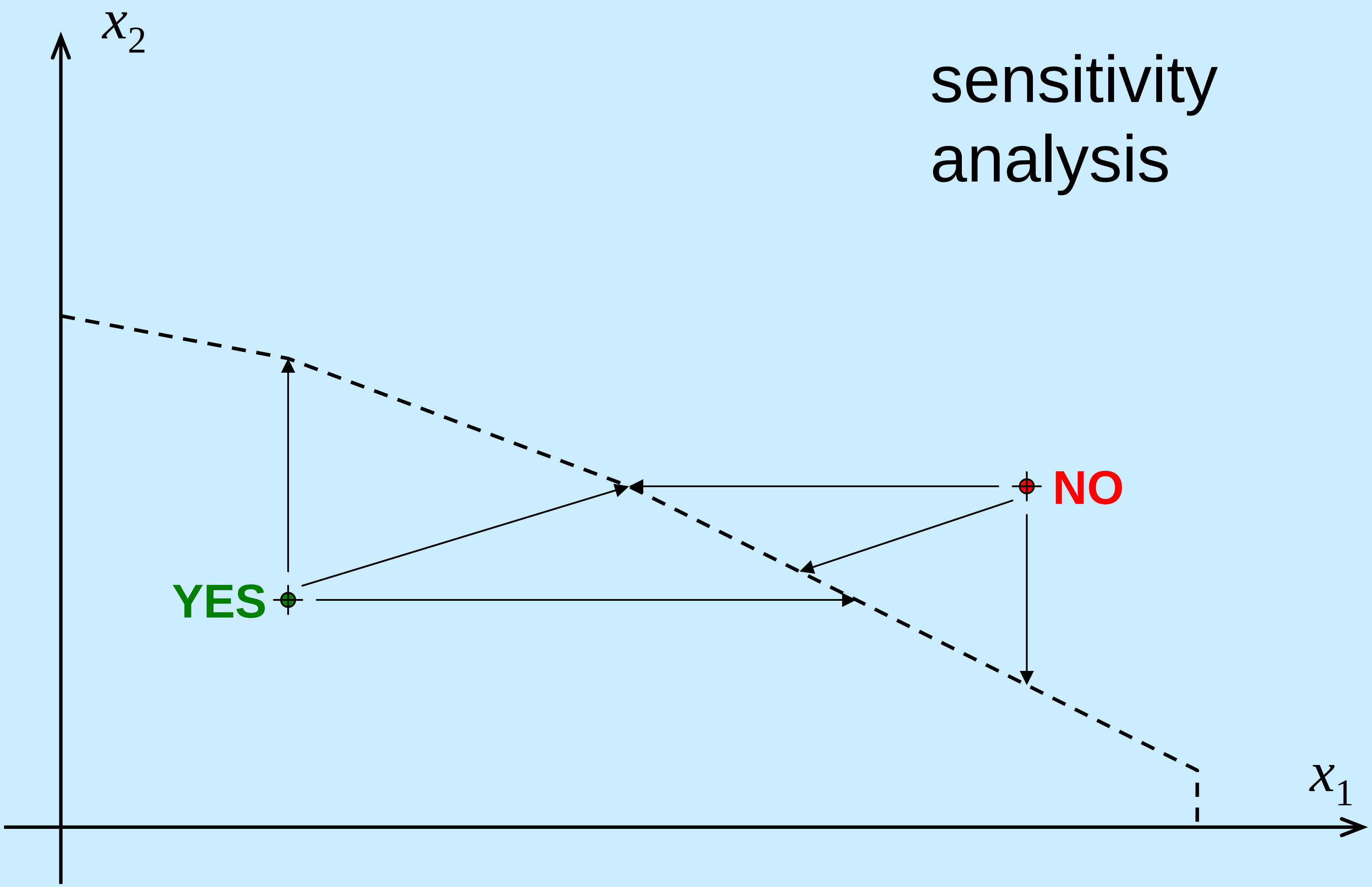
1. application developer provides the cost (utility) function
2. real-time analyst formulates the schedulability constraints for the given computing resources
3. an automated tool returns the best settings for the variables

Space of variables

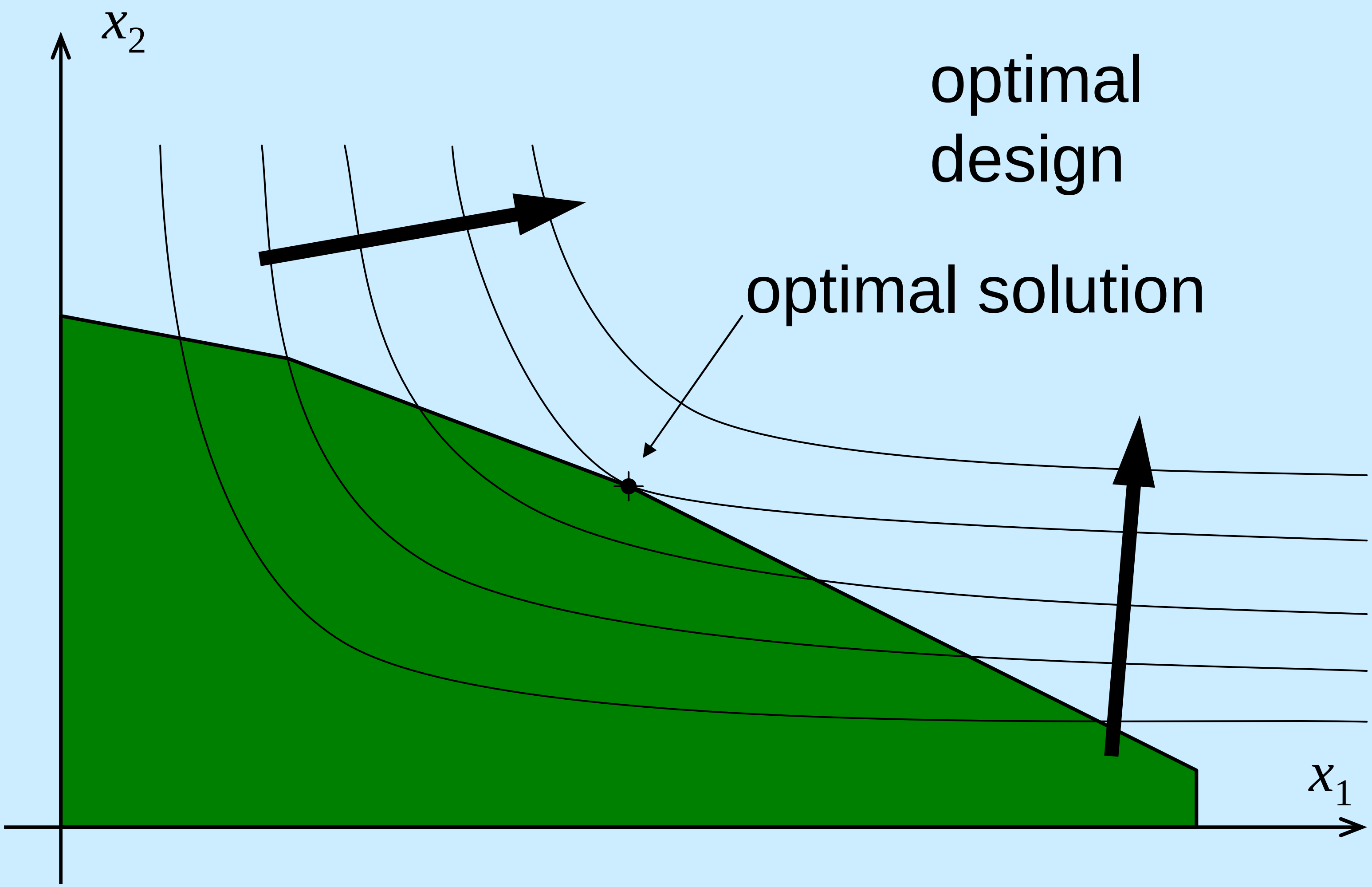


schedulability analysis

Space of variables



Space of variables

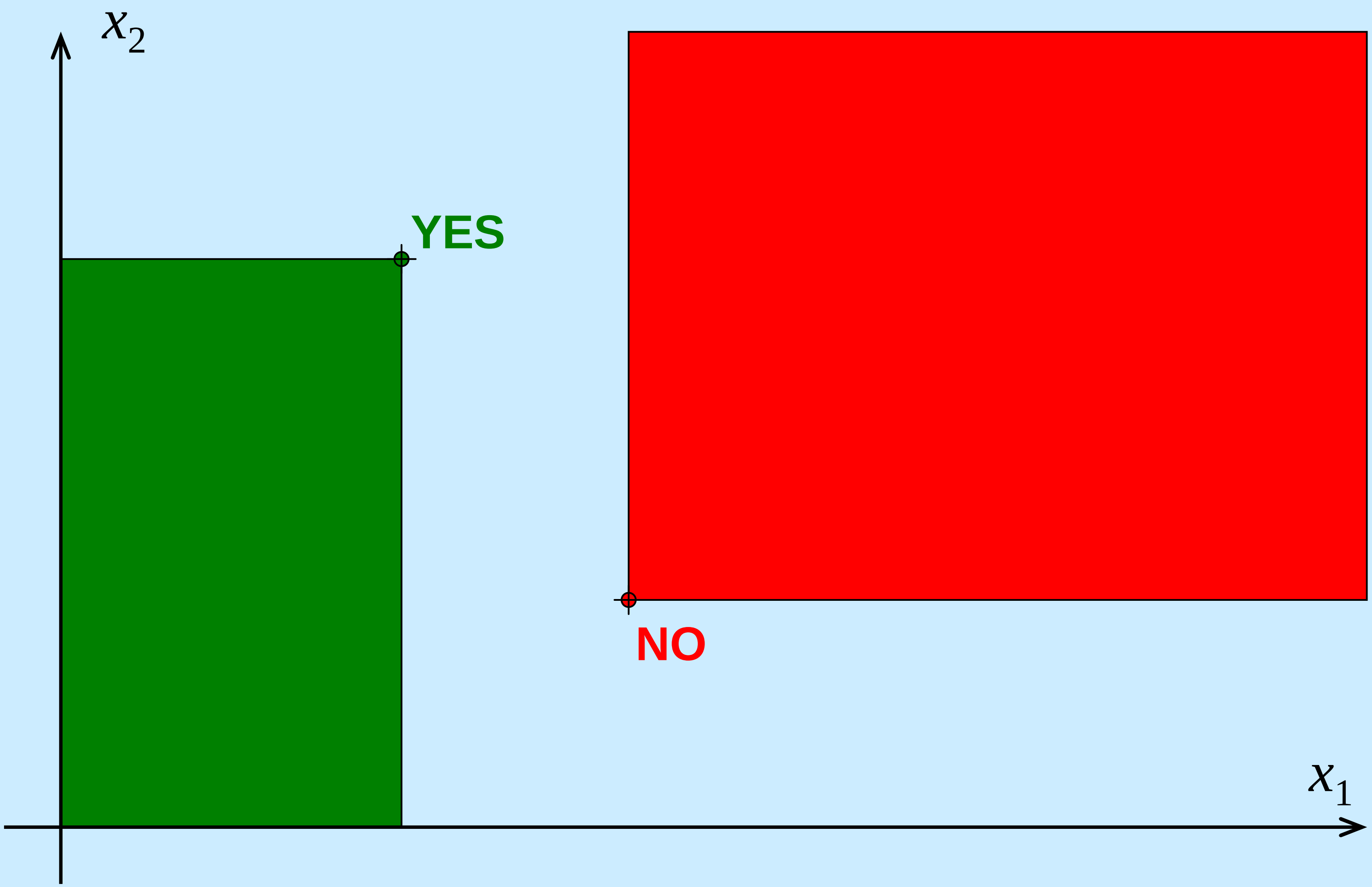


Notion of sustainability

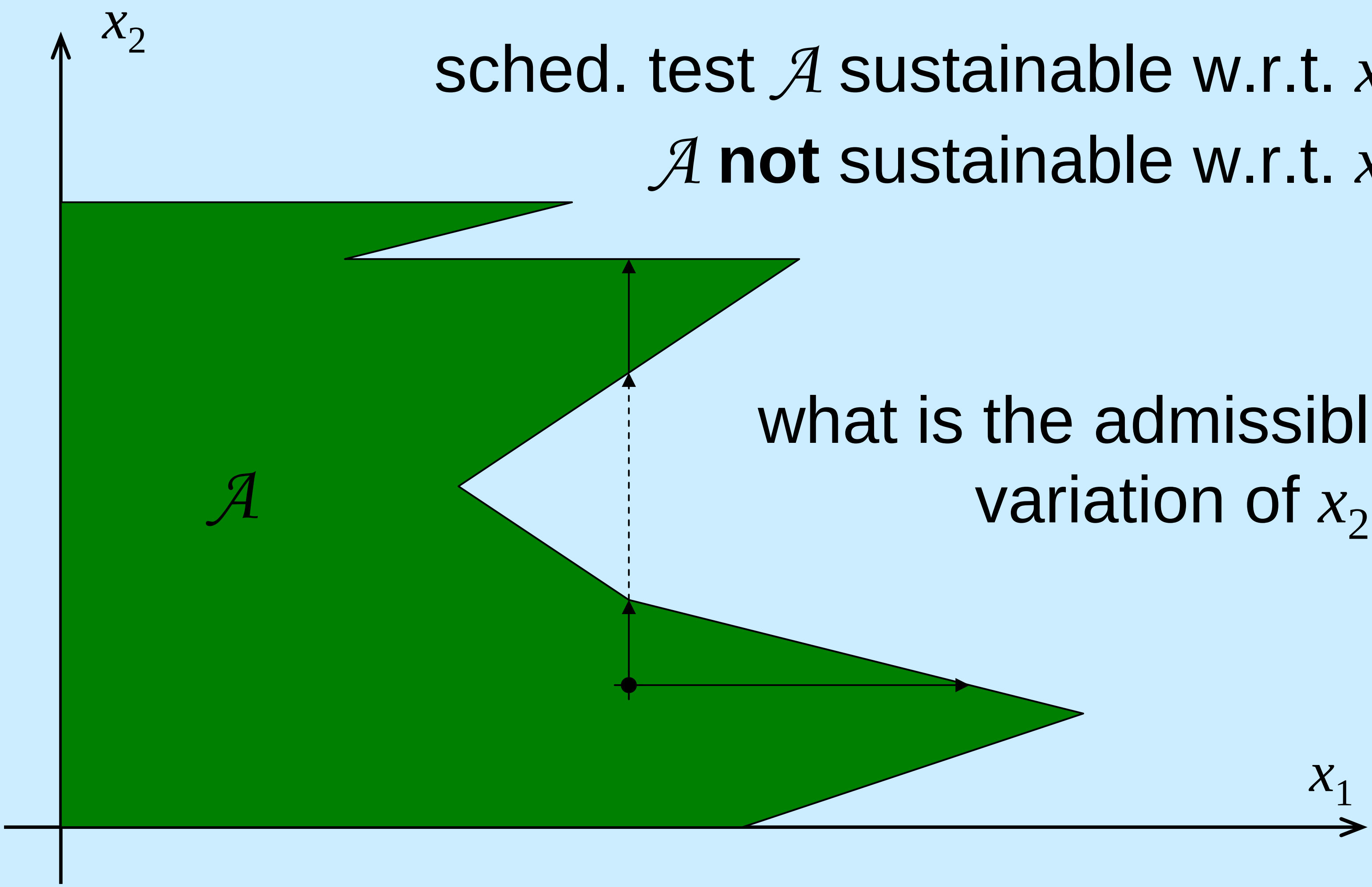
[Baruah, Burns @ RTSS06] A schedulability test is **sustainable** if any system deemed schedulable by the test remains schedulable when it has “looser constraints”.

Looser constraints: smaller computation times, longer period, longer deadline, smaller jitter, faster processor

Viewing the sustainability



Sensitivity and sustainability

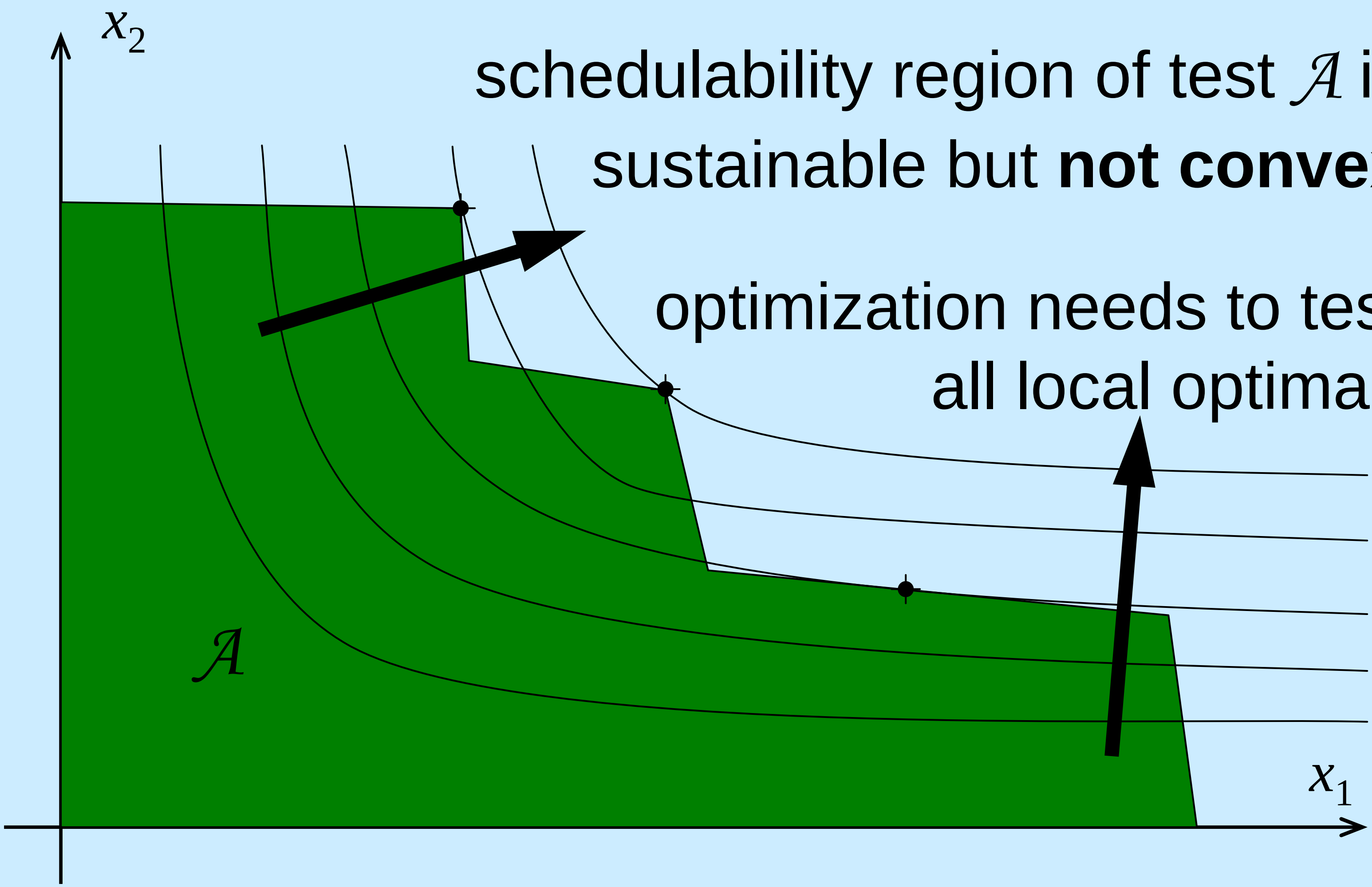


sched. test $\mathcal{A}$ sustainable w.r.t. x_1

$\mathcal{A}$ **not** sustainable w.r.t. x_2

what is the admissible variation of x_2?

Optimal design and convexity



schedulability region of test $\mathcal{A}$ is sustainable but **not convex**

optimization needs to test all local optima!!

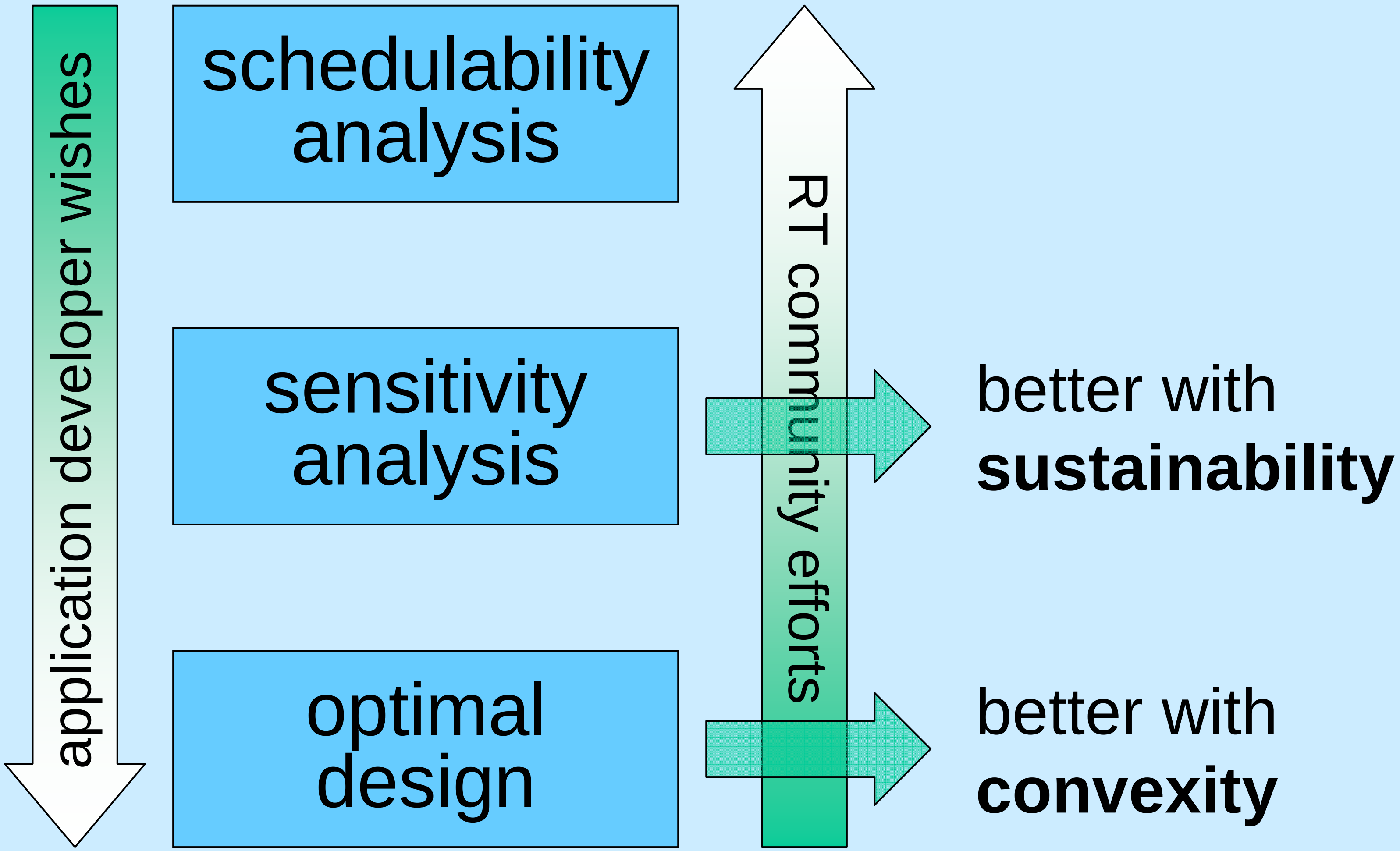
Convexity of popular tests

Uniprocessor scheduling algorithms are all sustainable.

What about convexity?

	when variables are...		
	C_i	T_i	D_i
utilization-based tests ($D_i = T_i$)			
exact DM			FP trivial DM ??
exact EDF ($D_i \neq T_i$)			

Summary



application developer wishes
schedulability analysis
sensitivity analysis
optimal design
RT community efforts
better with **sustainability**
better with **convexity**

References

Schedulability analysis

[Liu, Layland, 1973] First utilization based schedulability test

[everybody, everytime] Extensions to task models, scheduling algorithm, computing platform,...

Sensitivity analysis

[Vestal 1994] FP, comp times

[Punnekkat, Davis, Burns, 1997] FP, binary search

[Bini, Di Natale, Buttazzo, 2006] FP, comp times, periods

[Racu, Hamann, Ernst, 2006] FP, distributed task set

[Hoang, Buttazzo, 2006] EDF, deadlines

[Balbastre, Ripoll, Crespo, 2009] EDF, periods, deadlines

[George, Hermant, 2009] EDF, comp times

References

Optimal design

[Seto et al, 1996] optimal periods on utilization bound

[Aydin et al, 2001, "reward-based"] optimal comp times

[Bini, Di Natale, 2005] optimal periods on exact FP

[Wu, Bini, Buttazzo, 2008] EDF subopt convex deadlines

Sustainability/Convexity

[Baruah, Burns, 2006] def. sustainable analysis

[Hermant, George, 2009] convexity of EDF C-space

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Optimal design in control systems

Control systems are well suited for the optimal design:

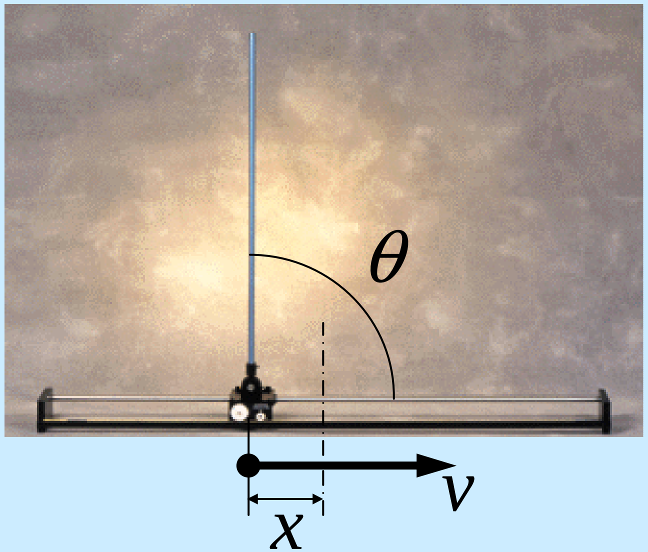
- very stable computation time
  - often controllers are just a multiplication by a matrix (no if statement)
- the cost can be measured quantitatively
  - as function of the state and the input

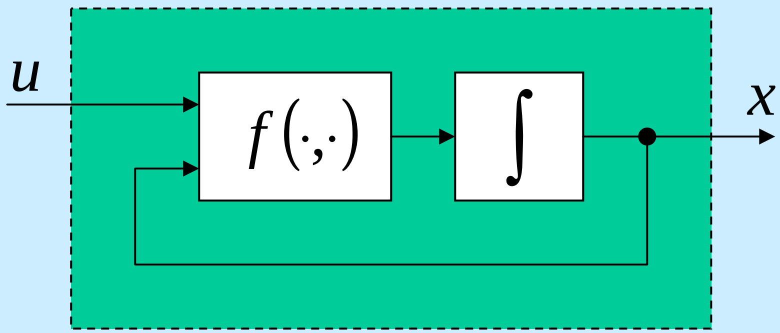
Introduction on control systems

- state of the plant x
  must reach stability (x=0)
- input to the plant u
- system dynamics
  differential equation

$$\begin{cases} \dot{x} = f(u, x) \\ x(0) = x_0 \end{cases}$$

The inverted pendulum





The cost in control systems

A classic expression of the cost is:

$$J = \int_0^\infty q\|x(t)\|^2 + \|u(t)\|^2 \,\mathrm{d}t$$

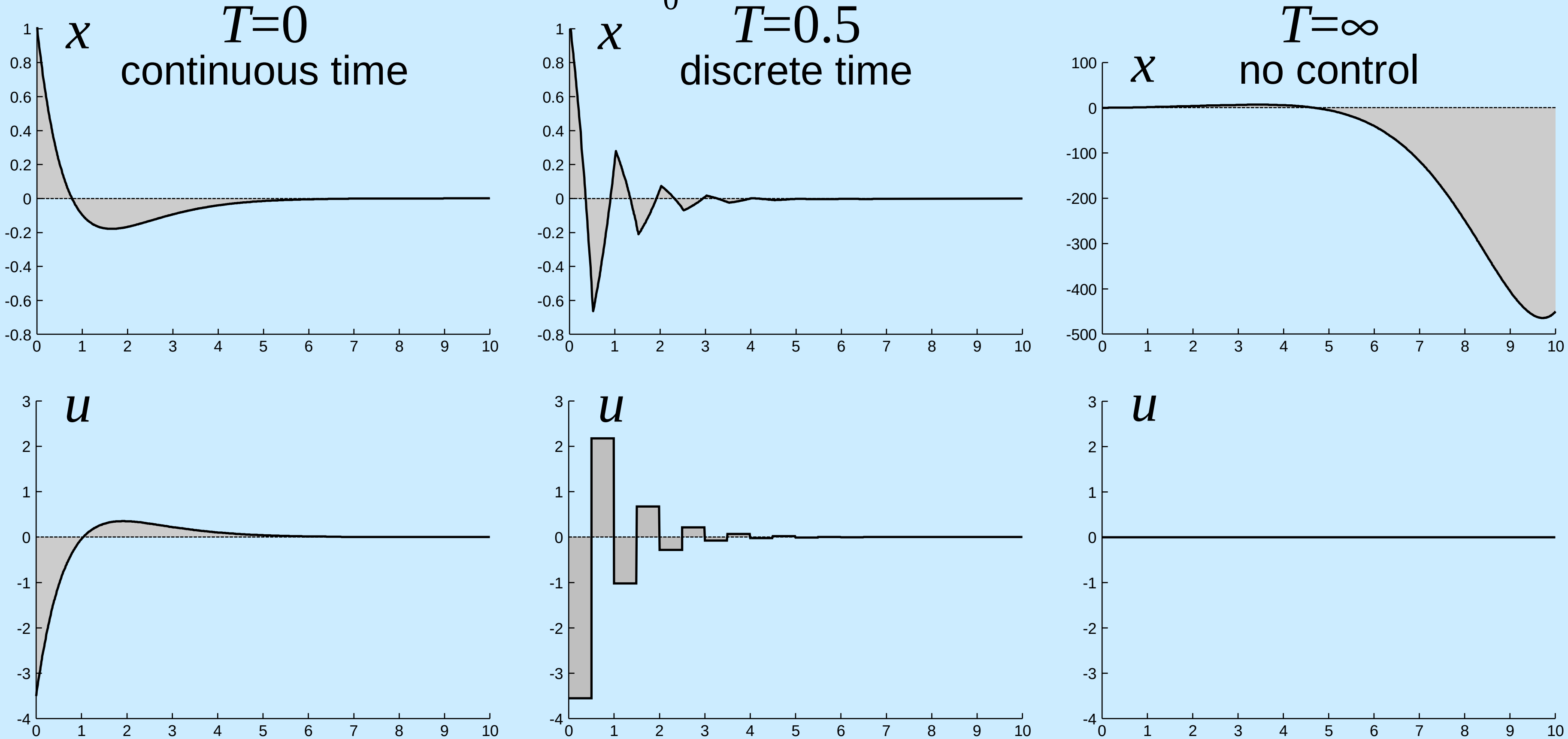
Remember: stability $\Rightarrow \lim_{t\to\infty} x(t) = 0$

q weights the relative importance of the state x over the input u:

- large q means we target fast convergence (of x)
- small q means we target little control action u

Cost vs. period (T)

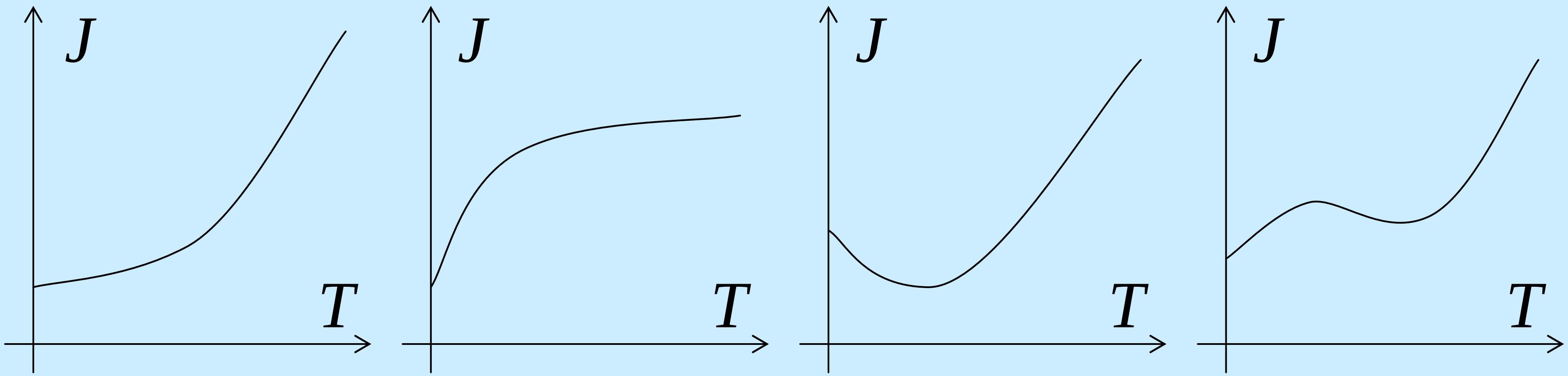
The cost J is $$J = \int_0^\infty \|x(t)\|^2 + \|u(t)\|^2 \,\mathrm{d}t$$



$$J(0) \le J(0.5) \le J(\infty) = \infty$$

Examples of cost $J(T)$

Classic cost functions are:



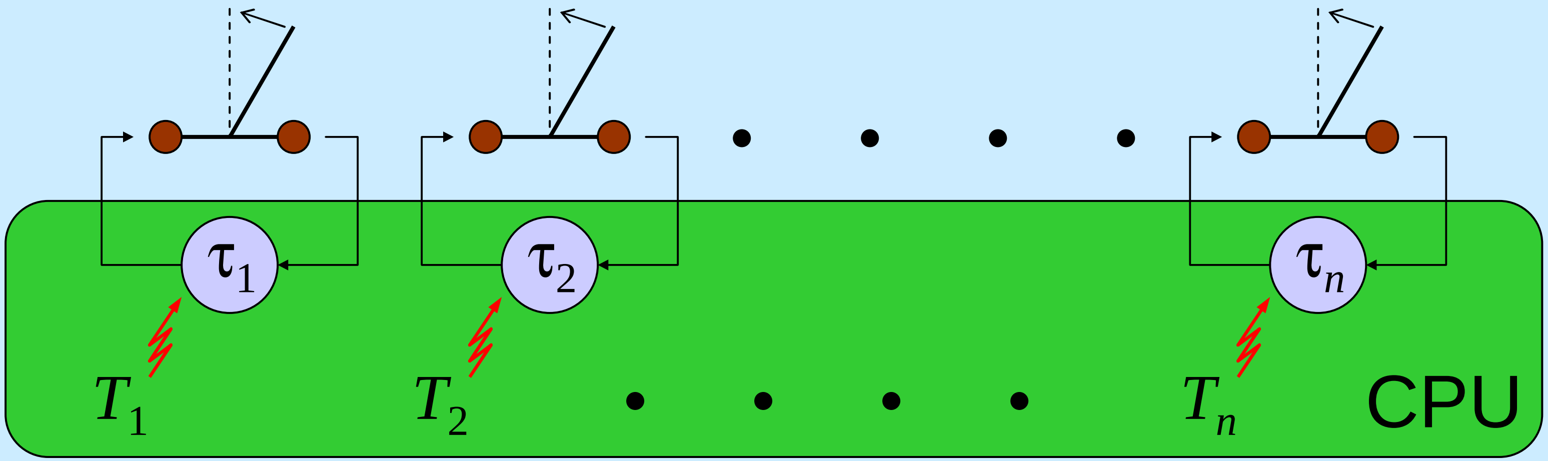
depending on:

- the weight q of the state x w.r.t. the input u
- the system dynamics f

Period assignment

The period (T) should as short as possible

However:

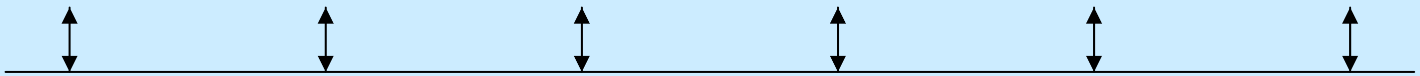


- n independent controllers with different periods ($T_1,\ldots,T_n$)
- the controllers run on the same CPU
- classic goal: minimize $\sum_{i=1}^{n} w_i T_i$

Scheduling models for digital control systems

A task schedule is not the only period...

1. sampling and actuation are simultaneous and strictly periodic (variable: period T_i)



2. sampling and actuation are separated by a constant delay (variables: period T_i, delay Δ_i)

3. actuations occur periodically with a jitter (variables: period T_i, delay Δ_i, jitter J_i)

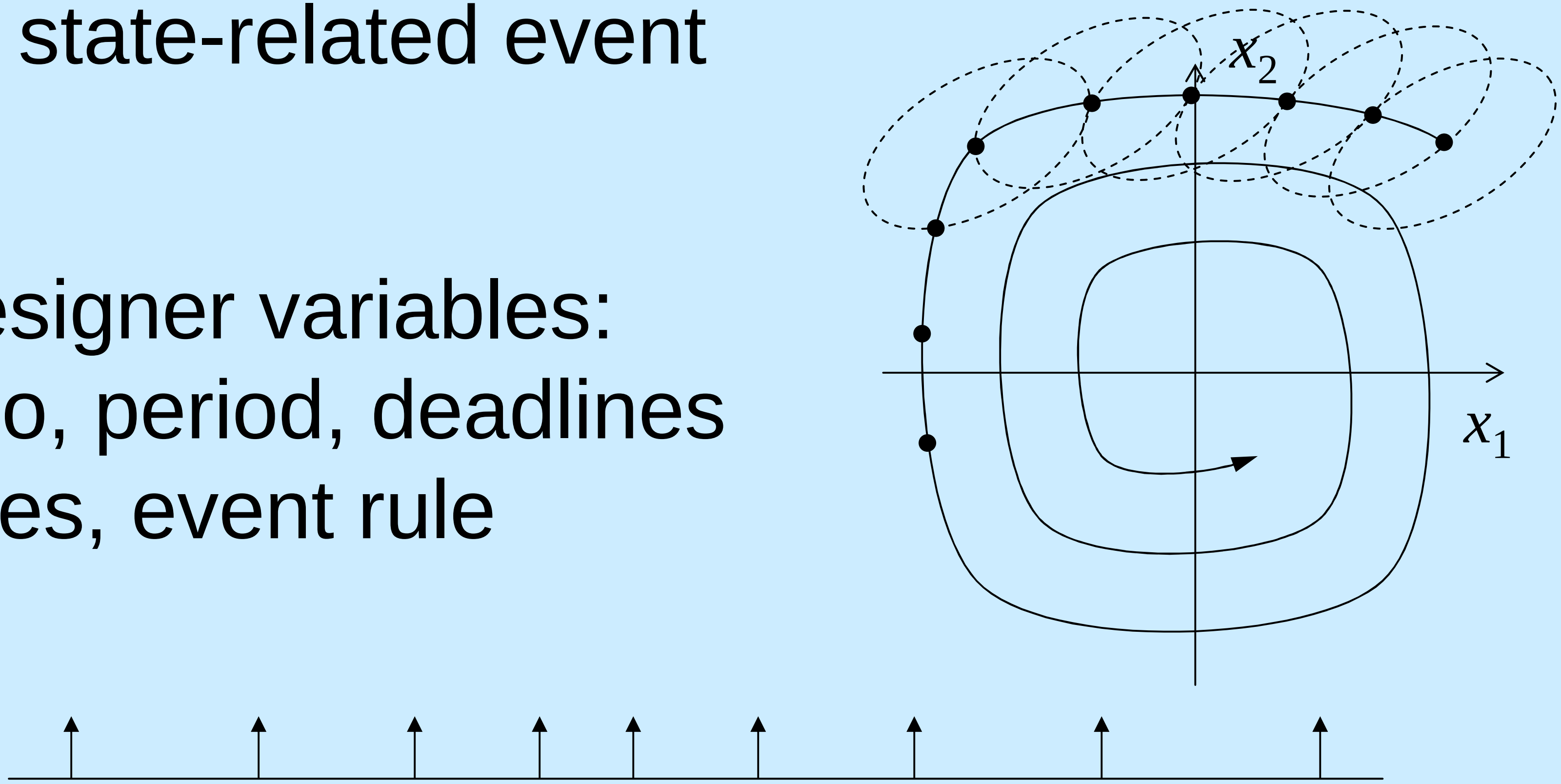
Event-driven sampling

Until now task activations are the variables

However the task may be activated based on state-related event

Designer variables:

- no, period, deadlines
- yes, event rule



References

[Diduch, Doraiswami, 1987] cost function from sampling period

[Seto et al, 1996] optimal periods, linear/exponential cost

[Cervin et al., 2004 “jitter margin”] amount of admissible output jitter

[Bini, Cervin, 2008] opt solution, linear cost (period, delay)

[Velasco, Martí][Anta, Tabuada][Wang, Lemmon] sched analysis of event-driven control tasks

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Ideas for the Future

The press-one-button machine

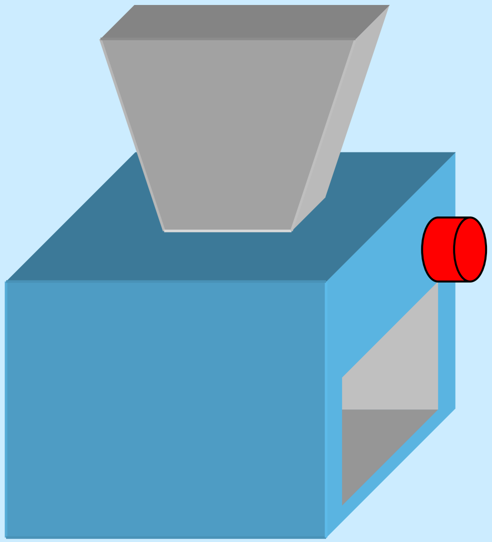
I want the best embedded system

What do you mean by "best"?

The one that maximise ...

What are your constraints?

50 Euro



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50 Euro

1. silicon
2. good theory