AN FPTAS FOR INTERFACE SELECTION IN THE PERIODIC RESOURCE MODEL

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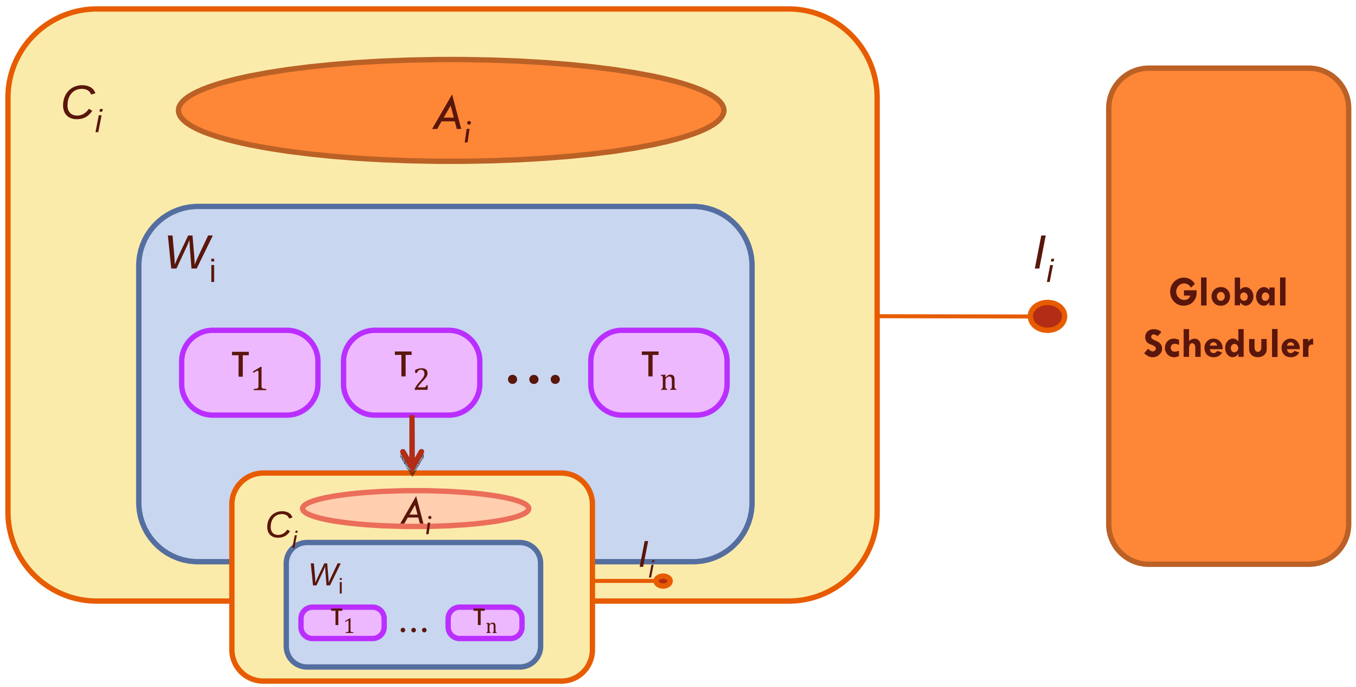
Overview

- *Setting:*
  - Compositional Real-Time Systems
  - Periodic Resources
- *Problem:* Interface Selection for Minimization of Interface Bandwidth (MIB-RT)
- *Motivation.*
  - *Subproblems*: Capacity Determination & Period Selection
- *Solution:* Parametric Approximation Algorithm

Setting: Compositional RTS

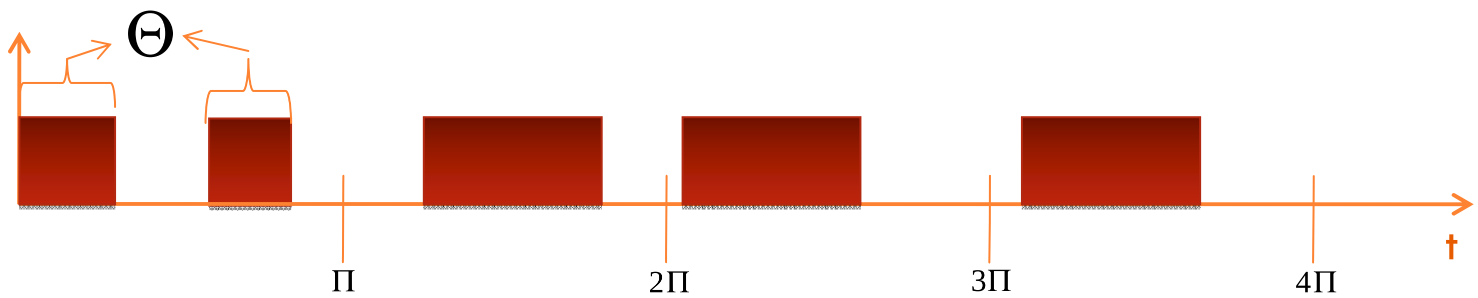
- Component C_i
  - Workload W_i
  - Component-Level Scheduling Algorithm A_i
  - Real-time Interface I_i



Setting: Periodic Resources

Periodic Resource Model

- Periodic resource, $\Gamma=(\Pi, \Theta)$ [Shin and Lee, RTSS03].
- Θ units of processing *capacity* in *period* Π.
- Assume $\Theta \leq \Pi$.



Interface Bandwidth

- Fraction of system's **resource supply** required by a component
- **Interference** of a component on other components
- For periodic resource: Θ/Π

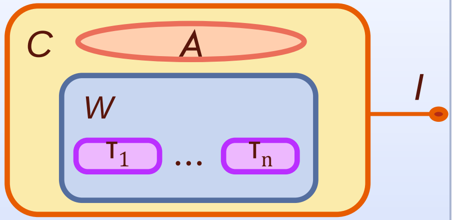
Problem: MIB-RT

Minimization of Interface Bandwidth (MIB-RT)

Given: Component C=(W, A)

Find: Interface I such that

- Workload W is A-schedulable upon component C with respect to interface I



Problem: Find interface **Θ** and **Π** that **minimizes interface bandwidth Θ/Π** while ensuring τ is EDF-Schedulable on Γ.

MIB-RT for a Periodic Resource:

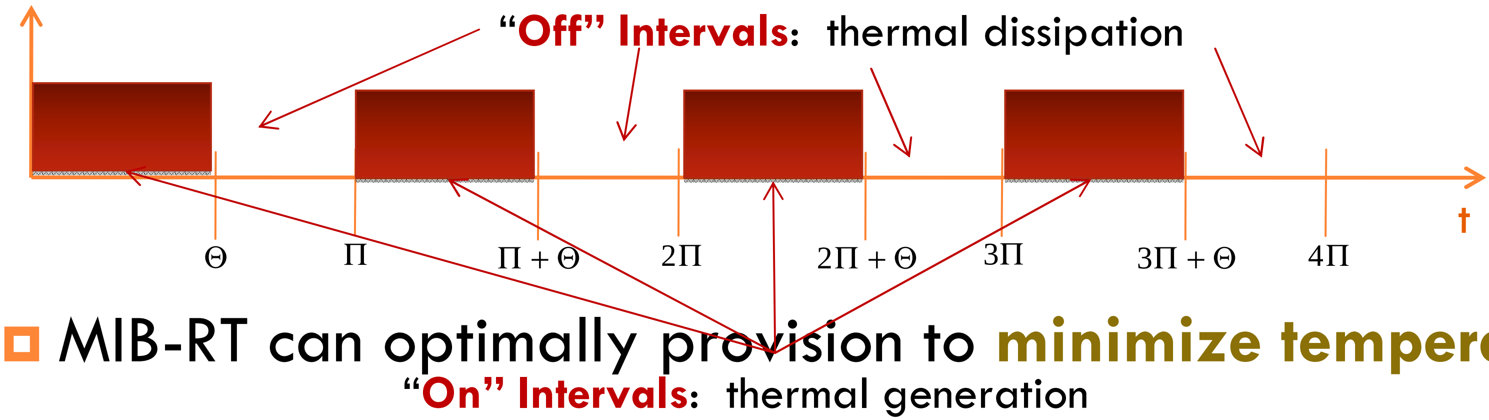
- ***W*** : Sporadic task system τ
- ***A*** : Earliest Deadline First (EDF)
- ***I*** : Periodic Resource parameters (i.e. Γ =(Π,Θ))

Motivation:
Why is MIB-RT Important?

- **Co-Scheduling of Components on Shared Processor.**
  - Resources $\Gamma_1 = (\Pi_1, \Theta_1)$, $\Gamma_2 = (\Pi_2, \Theta_2)$, …, $\Gamma_m = (\Pi_m, \Theta_m)$ may share a processor *if and only if*

    $$\Theta_1 / \Pi_1 + \Theta_2 / \Pi_2 + \ldots + \Theta_m / \Pi_m \leq 1.$$

  - Thus, MIB-RT promotes **composability**.
- **Thermal-Aware Scheduling**



  - MIB-RT can optimally provision to **minimize temperature**.

Task System

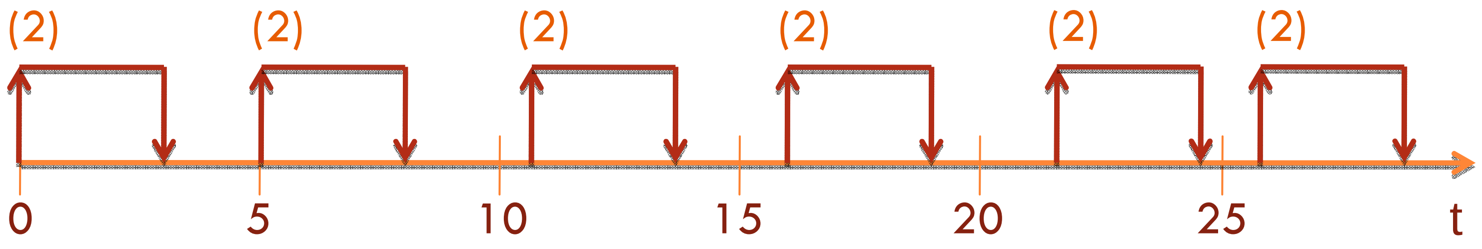
- Each component is a sporadic task system, $\tau = \{\tau_1, \tau_2 \ldots, \tau_n\}$

Sporadic Tasks

Characterized by the tuple $\tau_i = (e_i , d_i , p_i)$

- Worst case execution requirement, e_i
- *Task* Period, p_i
- Relative deadline, d_i

- Example: $\tau_1 = (2,3,5)$



Sub-Problems

- To solve MIB-RT, we need to address two sub-problems:
  1. **Capacity Determination.**
  2. **Periodic Selection.**

Sub-Problem: Capacity-Determination Algorithms

- **Capacity-Determination Algorithm $\mathcal{A}$:**
  1. Fix Π .
  2. Determine minimum capacity $\Theta(\mathcal{A}, \Pi, \tau)$ such that τ is EDF-schedulable upon resource $\Gamma = (\Pi, \Theta(\mathcal{A}, \Pi, \tau))$.

- Known Algorithms:
  - Sufficient, Linear-Time Algorithm [Shin and Lee, RTSS 2003].
  - Exact, Exponential-Time Algorithm [Easwaran et al., RTSS 2007].
  - Polynomial-Time Approximation Algorithm [Fisher and Dewan, ECRTS 2009].

Remaining Question: How do we **determine the optimal Π** given a capacity-determination algorithm?

Sub-Problem: Period Selection

Period-Selection Problem

Given:

- Sporadic Task System τ,
- Capacity-Determination Algorithm $\mathcal{A}$,
- Range of integer periods: $\Pi_{lower}, \ldots, \Pi_{upper}$

Find: Π' in range that minimizes $\Theta(\mathcal{A}, \Pi', \tau) / \Pi'$.

- **Observation:** Extreme values Π_{lower} and Π_{upper} do not always result in smallest bandwidth!
  - **Example:** $\tau = \{\tau_1 = (e_1, d_1, p_1) = (1, 301, 1000)\}$ with $\Pi_{lower} = 80$ and $\Pi_{upper} = 150$.

Minimum bandwidth when $\Pi = 100$!

Sub-Problem: Period Selection

- Easwaran [PhD Thesis, 2007] gives an <u>exact</u>, exponential-time algorithm which depends on Π_{upper}.
  - Does potentially exponential amount of work for each $\Pi \in \{\Pi_{lower}, \ldots, \Pi_{upper}\}$.
  - Intractable if $\Pi_{lower}, \ldots, \Pi_{upper}$ range is large.

Our Goal: Trade-off accuracy for huge reductions in computational complexity?.

Solution: Objectives

General Period-Selection Approximation

Input:

- Sporadic Task System τ
- Capacity-Determination Algorithm $\mathcal{A}$
- Range of integer periods: $\Pi_{lower}, \ldots, \Pi_{upper}$
- Accuracy Parameter ε: $0 < \varepsilon \leq 1$

Output: Periodic Resource $\Gamma = (\Pi', \Theta(\mathcal{A}, \Pi', \tau))$ such that τ is EDF-schedulable upon Γ and

$$\Theta(\mathcal{A}, \Pi^*, \tau)/\Pi^* \leq \Theta(\mathcal{A}, \Pi', \tau)/\Pi' \leq (1+\varepsilon)\cdot(\Theta(\mathcal{A}, \Pi^*, \tau)/\Pi^*)$$

Returned bandwidth.

Optimal band

$\mathcal{A}$ and

Approximation Ratio

Solution: Assumptions

- Capacity-determination algorithm $\mathcal{A}$ is **monotonically non-decreasing** in Π.
  - Definition:
    For all $\Pi_1 \leq \Pi_2$, then $\Theta(\mathcal{A}, \Pi_1, \tau) \leq \Theta(\mathcal{A}, \Pi_2, \tau)$.
  - All previously known capacity-determination algorithms are monotonically non-decreasing.

Solution:

Period-Selection Algorithm

Properties of Step 1:

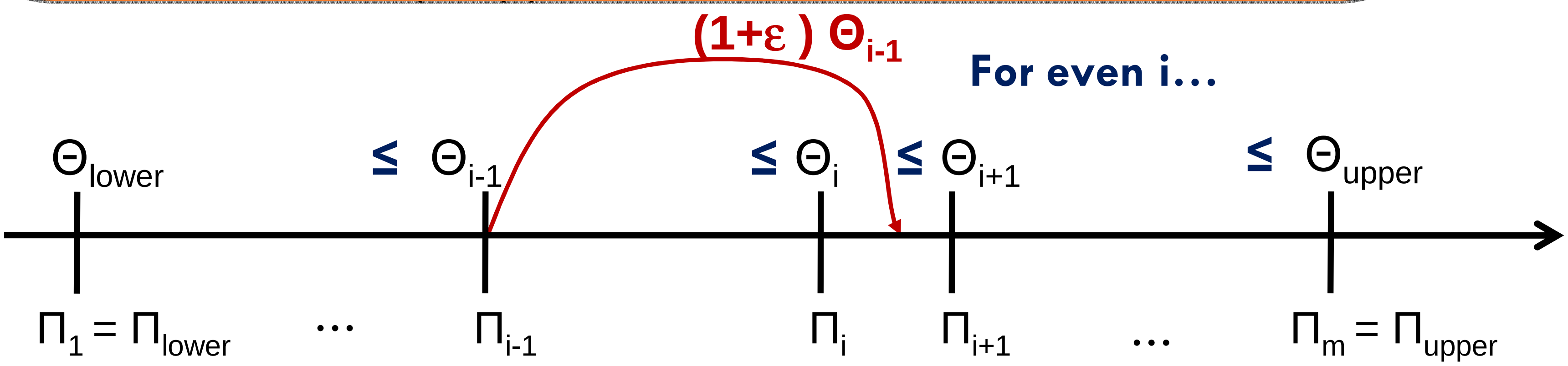
1. The size of m is at most:

   $$O(\log_{(1+\varepsilon)} (\Theta_{upper} / \Theta_{lower})).$$

2. *Binary search* used to find next Π_i in time:

   $$O(\log_2 \Pi_{upper}).$$

3. For any i: $1 < i < m$, either:
   - $\Pi_i = \Pi_{i-1} + 1$, or
   - $\Theta(\mathcal{A}, \Pi_i, \tau) \leq (1+\varepsilon) \cdot \Theta(\mathcal{A}, \Pi_{i-1}, \tau)$.



Solution:
Period-Selection Algorithm

Step 1: Select subset $\{\Pi_1, \ldots, \Pi_m\}$ of range $\Pi_{lower}, \ldots, \Pi_{upper}$ according to ε and $\mathcal{A}$.

Step 2: Evaluate $\Theta(\mathcal{A}, \Pi_i, \tau)/\Pi_i$ for all $\Pi_i \in \{\Pi_1, \ldots, \Pi_m\}$.

- Return $\Gamma = (\Pi', \Theta(\mathcal{A}, \Pi', \tau))$ that minimizes bandwidth.

Time Complexity:

$$O(T_{\mathcal{A}}(\tau) \cdot \log_2 (\Theta_{upper} / \Theta_{lower}) \cdot (\log_2 \Pi_{upper}) / \varepsilon)$$

Algorithm $\mathcal{A}$ Time Complexity

Solution:

Proof of Approximation Ratio

Theorem

If our algorithm returns $\Gamma = (\Pi', \Theta(\mathcal{A}, \Pi', \tau))$, then

$\Theta(\mathcal{A}, \Pi^*, \tau)/\Pi^* \leq \Theta(\mathcal{A}, \Pi', \tau)/\Pi' \leq (1+\varepsilon)\cdot(\Theta(\mathcal{A}, \Pi^*, \tau)/\Pi^*)$

- **Proof**: Obviously, $\Theta(\mathcal{A}, \Pi^*, \tau)/\Pi^* \leq \Theta(\mathcal{A}, \Pi', \tau)/\Pi'$.

  So, we will show second inequality.

Consider subset $\{\Pi_1, \ldots, \Pi_m\}$.

Two Cases:

1. Π^* is in $\{\Pi_1, \ldots, \Pi_m\}$. $\Rightarrow \Gamma^* = (\Pi^*, \Theta(\mathcal{A}, \Pi^*, \tau))$ is returned.
2. Π^* is <u>not</u> in $\{\Pi_1, \ldots, \Pi_m\}$.

Solution:
Proof of Approximation Ratio

Theorem

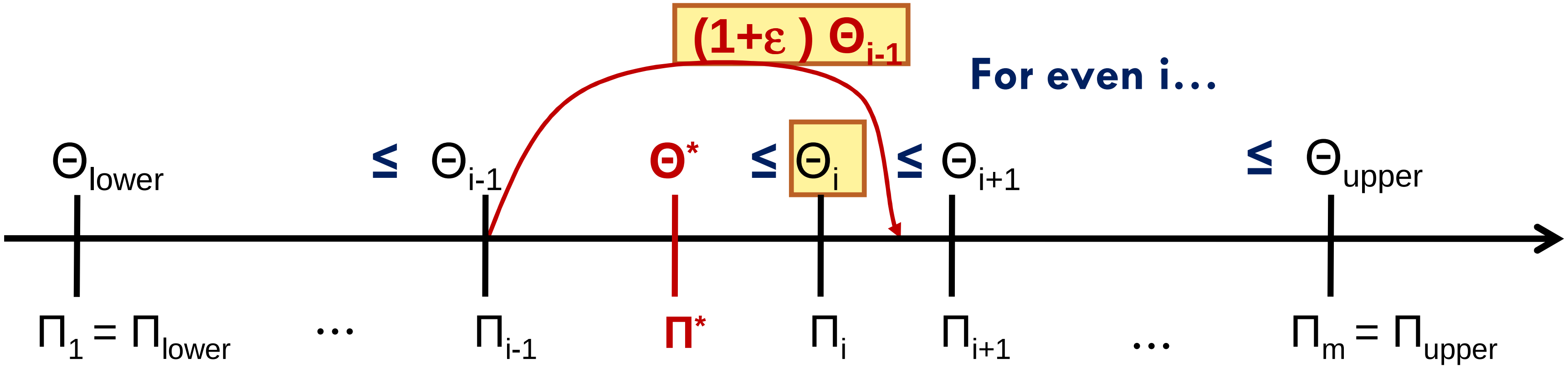
If our algorithm returns $\Gamma = (\Pi', \Theta(\mathcal{A}, \Pi', \tau))$, then

$\Theta(\mathcal{A}, \Pi^*, \tau)/\Pi^* \le \Theta(\mathcal{A}, \Pi', \tau)/\Pi' \le (1+\varepsilon)\cdot(\Theta(\mathcal{A}, \Pi^*, \tau)/\Pi^*)$

- **Proof**(con't): Π^* is not in $\{\Pi_1, \ldots, \Pi_m\}$.

$\Theta(\mathcal{A}, \Pi_{i-1}, \tau)/\Pi_i \le \Theta(\mathcal{A}, \Pi^*, \tau)/\Pi^*$.

$\Rightarrow (1+\varepsilon)\cdot \Theta(\mathcal{A}, \Pi_{i-1}, \tau)/\Pi_i \le (1+\varepsilon)\cdot\Theta(\mathcal{A}, \Pi^*, \tau)/\Pi^*$. ■



Solution:

Combine with Capacity Approximation

Fully Polynomial-Time Approximation Scheme for Capacity Determination (FPTAS-CD)
[Fisher & Dewan, ECRTS 2009]

Given sporadic task set τ, fixed period Π, and accuracy parameter ε returns Θ(FPTAS-CD, Π, τ, ε) such that

- $\Theta(\text{FPTAS-CD}, \Pi, \tau, \varepsilon) \leq (1+\varepsilon)\cdot\Theta(\text{OPT}, \Pi, \tau)$
- Complexity $O(n \lg n/\varepsilon)$

Optimal Exact Capacity Algorithm

Solution:
Combine with Capacity Approximation

τ

$\Pi_{lower}, \ldots, \Pi_{upper}$

ε

$\mathcal{A}$ = FPTAS-CD

Overall Period Selection Approximation

$\Gamma = (\Pi, \Theta)$

Note: General algorithm was w.r.t. arbitrary $\mathcal{A}$.

Theorem

The above algorithm returns $\Gamma = (\Pi, \Theta)$ with

$$\Theta(OPT, \Pi^*, \tau)/\Pi^* \leq \Theta/\Pi \leq (1+\varepsilon)\cdot(\Theta(OPT, \Pi^*, \tau)/\Pi^*)$$

in time complexity

$$O((n \lg n)\cdot (\lg^2 \Pi_{upper} + (\lg \Pi_{upper})(\lg p_{max}))/\varepsilon^2)$$

Max task period in τ.

FPTAS: Polynomial in input size and 1/ε

Conclusions

- *Focus:* MIB-RT problem selecting for periodic resource interfaces.
- *Algorithms:* *Parametric Approximation Algorithm* for *period* and *capacity* selection.
  - General approach with respect to a given algorithm.
  - Specific approach using previous algorithm for an overall approximation to optimal interface parameters.
- *Analysis:* Given ε, the algorithm runs in polynomial in terms of input size and 1/ε and has approximation ratio (1+ ε):
  - Fully Polynomial Time Approximation Scheme(FPTAS)
- *Future work:* Applying compositional ideas to provisioning processing time for thermal-aware systems.

Thank You

Questions?

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