LRE-TL: An Optimal Multiprocessor Scheduling Algorithm for Sporadic Task Sets

Shelby Funk

Vijaykant Nadadur

The University of Georgia

Multiprocessor real-time systems

- Increasingly, real-time systems are developed on multiprocessors
  - Many popular uniprocessor real-time scheduling algorithms do not perform well on multiprocessors
- Optimal multiprocessor real-time scheduling algorithms suffer from one of two shortcomings
  - They impose high system overhead
  - They apply to restricted processing models

Task model

- Periodic & sporadic tasks: A mechanism for executing jobs repeatedly
- $T_i = (\phi_i, p_i, e_i)$
  - ϕ_i = offset
    - Periodic task: release time of first job
    - Sporadic task: earliest release time of first job
  - p_i = period
    - Periodic task: Exact time between jobs
    - Sporadic task: Minimum time between jobs
  - e_i = worst-case execution time
- If a task T_i generates a job at time a_i, then it must be allowed to execute for e_i time units during the interval $[a_i, a_i+p_i]$

Task utilization

- Given a periodic or sporadic task $T_i=(p_i,e_i)$, the utilization of T_i is e_i/p_i
  - Average proportion of processing time this task will require
- Given a set of tasks $\tau = \{T_1, T_2, \ldots, T_n\}$
  - $U(\tau)$ = τ's total utilization = $\Sigma\,(e_i/p_i)$
  - u_{max} = τ's maximum utilization = $\max\{e_i/p_i\}$
- Many tests are based on task utilization
  - τ can be scheduled on an m-processor identical multiprocessor iff $U(\tau) \leq m$ and $u_{max} \leq 1$

Online multiprocessor scheduling

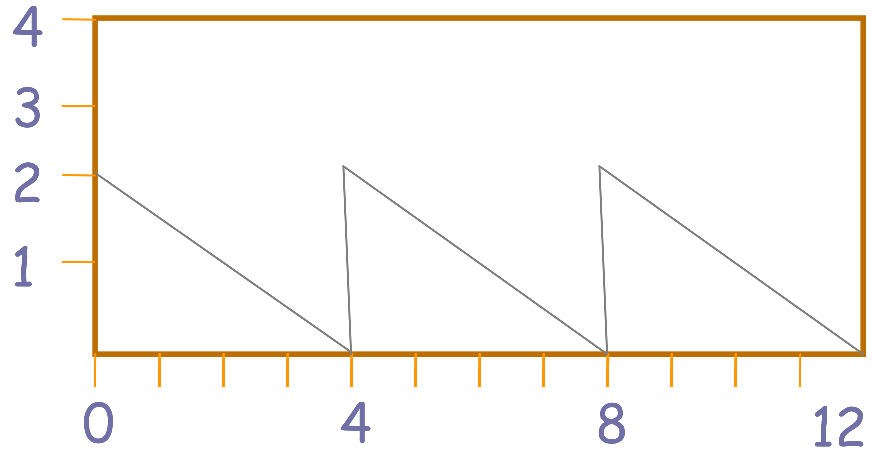
- The utilization test states *some* schedule exists
  - This schedule might be difficult to implement
- Hong and Leung proved that no online scheduling algorithm can be optimal when deadlines are not all equal

Ideal (but impractical) schedule

- Ideally, we would execute all tasks at a constant rate
  - Example $T_1 = (2,4)$, $T_2 = (3,5)$, and $T_3 = (6,8)$

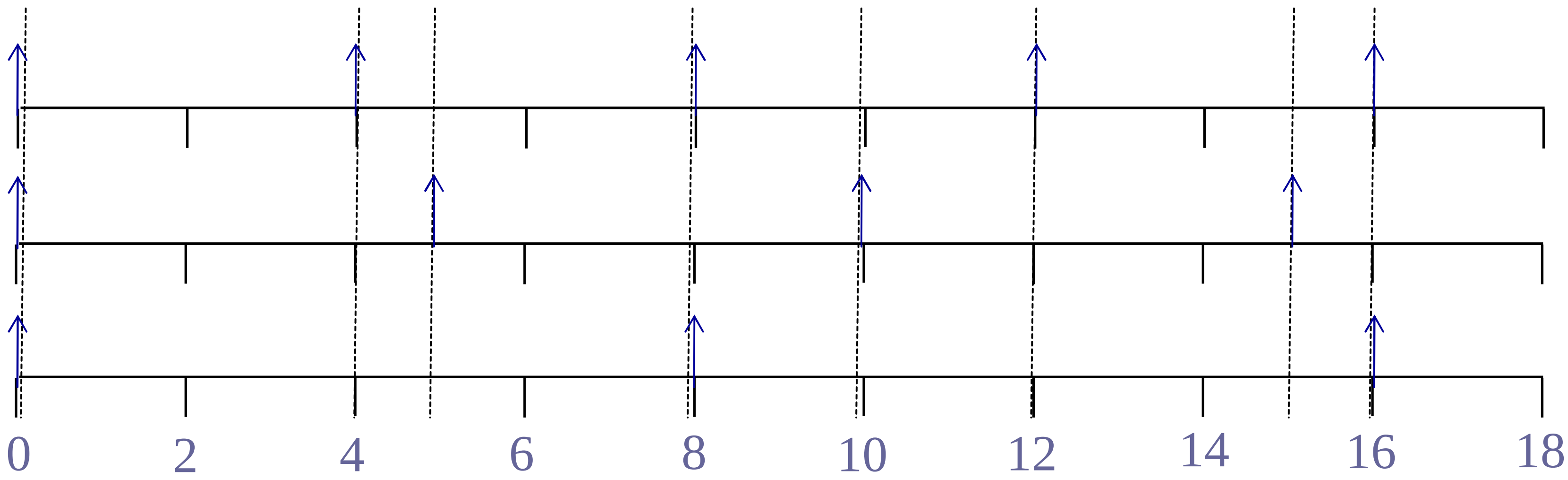
Proportion Processor per Task

T1 Remaining Execution



LLREF overview

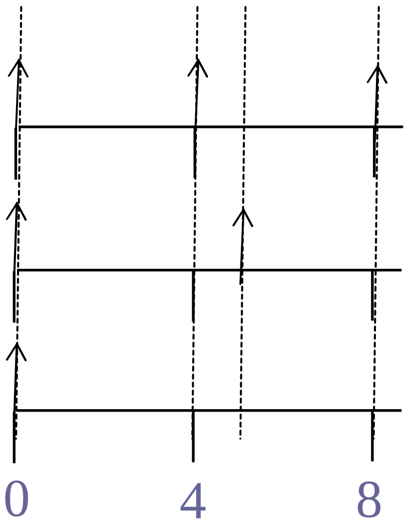
- The timeline is broken into TL planes
  - Time and Local execution time
  - Dividing points are determined by task deadlines
  - Scheduling within a TL plane $[t_{i-1}, t_i]$ ensures tasks have executed at their ideal amount at by time t_i

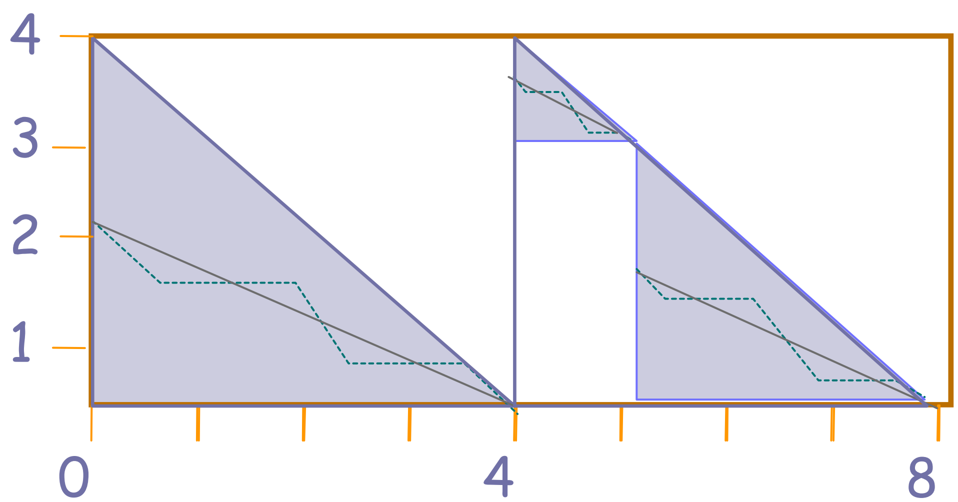


- **Example** $T_1 = (4,2)$, $T_2 = (5,3)$, and $T_3 = (8,6)$

LLREF schedules

- Tasks are guaranteed to have executed at ideal rate on every deadline
  - Hence, all deadlines are met – LLREF is optimal for multiprocessors





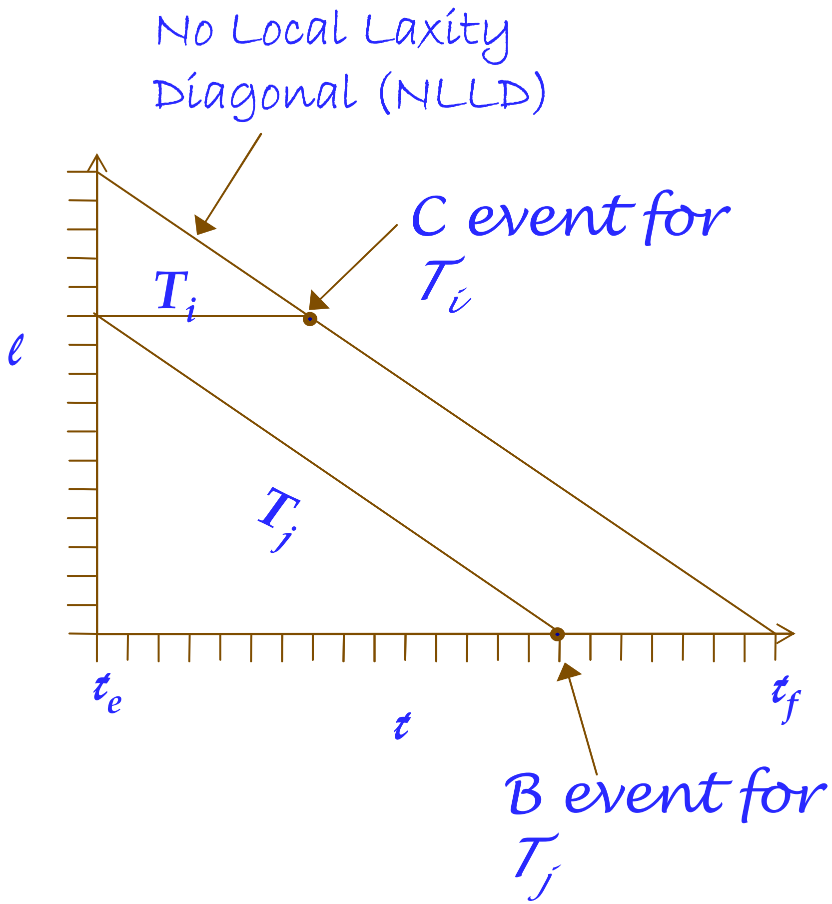
T_1 Remaining Execution Actual and Ideal in first 3 TL planes

Scheduling TL planes

- Each task is assigned a local workload for the given TL plane $[t_e, t_f)$
- ℓ_i = remaining time for T_i in current TL plane
  - At start of each TL plane $\ell_i = u_i \cdot (t_f - t_e)$
- r_i = local utilization within TL plane
  - $r_i = \ell_i / (t_f - t_{cur})$
- R_t = total local utilization at time t
- The m tasks with largest local remaining execution time are selected to execute
  - Tasks are scheduled until a B or a C event

Scheduling events

- Within the plane, each task is represented with a *token*
  - A correct schedule keeps all tokens within the plane
- A new scheduling event occurs under two conditions
  - B (bottom) events: A task's token hits the bottom of the TL plane
  - C (critical) events: A task's token hits the NLLD

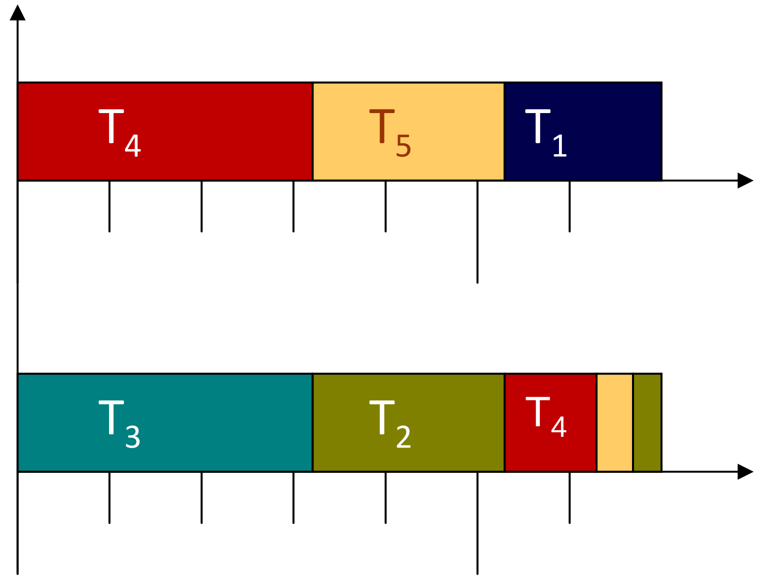


LLREF scheduler

- At every scheduling event, the m tasks with the largest local execution execute until a B or a C event occurs
  - LLREF = Largest Local Remaining Execution First

Example

$T_1=(7,2)$, $T_2= (10,3)$, $T_3= (9,4)$,
$T_4 = (12,7)$, $T_5 = (14,5)$



t=0		t=3.1		t=5		t=6		t=6.6		t=6.8		t=7
ℓ	B/C	ℓ	B/C	ℓ	B/C	ℓ	B/C	ℓ	B/C	ℓ	B/C	ℓ
2.0	5.0	2.0	1.9	2.0	2.0	1.0	1.0	0.4	0.4	0.2	0.2	0
2.1	4.9	2.1	2.1	0.2	1.8	0.2	0.8	0.2	0.2	0	--	0
3.1	3.1	0	--	0	--	0	--	0	--	0	--	0
4.1	4.1	1.0	2.9	1.0	1.0	0	--	0	--	0	--	0
2.5	4.5	2.5	2.5	0.6	1.4	0.6	0.6	0	--	0	--	0

LLREF complexity

- When a B or C event occurs the following steps are taken
  - Remaining execution time of m executing tasks are updated O(m)
  - Tasks are sorted by local remaining execution time O(n)
  - The m tasks with highest remaining execution time are selected to execute O(1)
  - The earliest upcoming B and C events are determined O(1)
  - Scheduling proceeds until that event

Reducing overhead

- Recall Hong and Leung's claim: No online scheduling algorithm can be optimal when deadlines are not all equal
  - When deadlines are equal, there only two concerns
    - Ensuring jobs with zero laxity execute immediately
    - Ensuring jobs with zero remaining execution do not execute
- LLREF makes all deadlines equal within a TL plane
- We can remove the sort step from the scheduling algorithm

Theorem

- Assume that τ is executing on m processors and the following conditions hold at time t
  - The total local utilization of τ is at most m
  - The maximum local utilization of τ is at most 1
  - There are m tasks with non-zero remaining local execution time
- If any m tasks execute and no task's local execution time becomes negative, then local utilization will not increase over time

Modified task maintenance

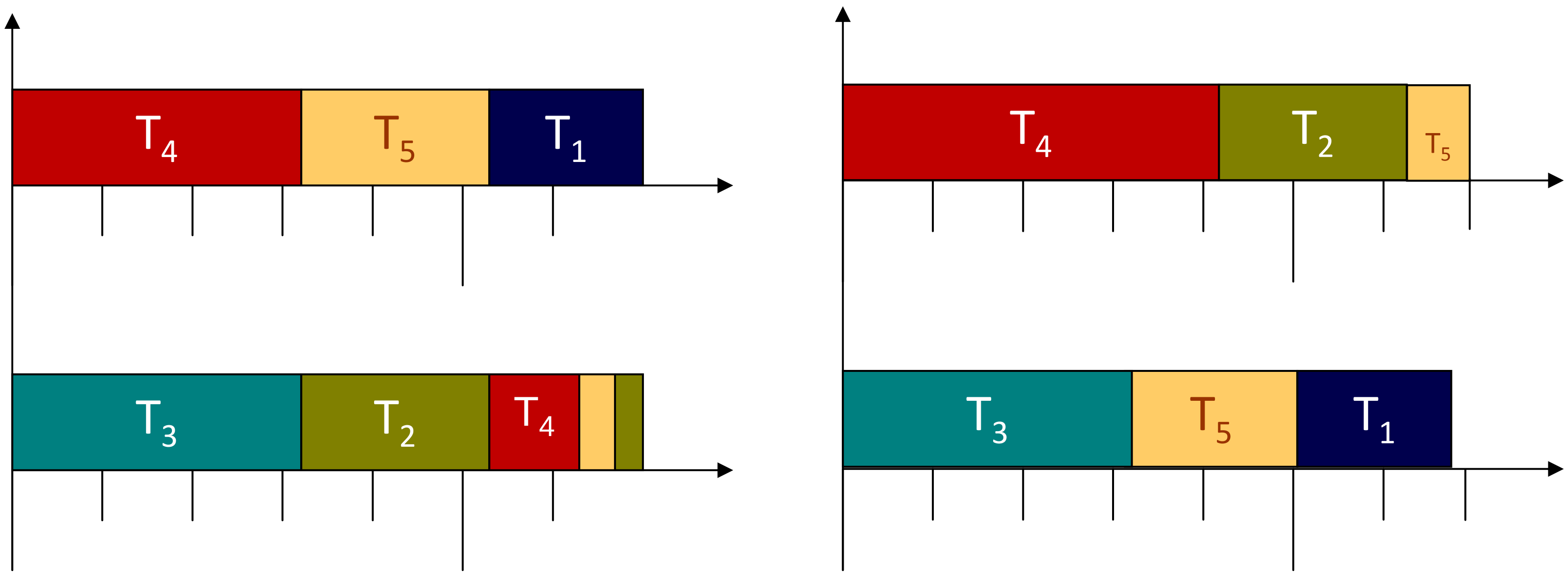
- Two min heaps: B-heap and C-heap
  - key = Amount of time to a B/C event
- When an event occurs, preempt as few tasks as possible
  - No preemption for B events – simply select any non-executing task to execute
  - Preempt one task for C event – schedule the zero laxity task on the freed processor

New scheduling event

- The theorem observed that total local utilization does not decrease over time
- If a sporadic task generates a job within a TL plane, we invoke an *A* event
  - Let $\ell_i = u_i \cdot (t_f - t_{cur})$, where t_f = end of TL plane and t_{cur} = arrival time
  - Add T_i to the non-executing heap
    - If $u_i = 1$, T_i must preempt some executing task
- Total local utilization at *A* event is at most $m - u_i$ and T_i's local utilization = u_i
  - Total utilization cannot exceed m

Two schedules

- Compare the schedules for T_1=(7,2), T_2= (10,3), T_3= (9,4), T_4= (12,7), T_5= (14,5)



Algorithm comparison

	LLREF	LRE-TL
Number of B/C events per TL plane	O(n)	O(n)
Max number of preemptions during each event	O(m)	O(1)
Running time per B/C event	O(n)	O(log m + log (n-m))
Running time per A event	--	O(log (n-m))

Conclusion

- Introduced method to reduce complexity of LLREF
- Introduced method to allow for sporadic task arrivals
- In future, we hope do the following
  - Reduce complexity even further
  - Apply results to uniform multiprocessors
  - Incorporate resource sharing
  - Apply to DVFS multiprocessing

A-event special case

- What if T_i invokes a job with deadline d_i, where $d_i < t_j$?
  - Deadline within a TL-plane is not allowed!
- Split remainder of TL-plane into two pieces
  - $[t,d_i)$ and $[d_i,t_j)$
- T_i executes for e_i time units during $[t,d_i)$
- Any other task T_j has $\ell_{j,t}$ divided into two pieces
  - Size of pieces proportional to length of the two intervals